

Binary operation: -

Let A be a non-empty set. Any function from $A \times A$ to A i.e. combines two elements of a set to produce another element of the same set is called binary operation. It is denoted by $\circ: A \times A \rightarrow A$ or $f: A \times A \rightarrow A$

If $f: A \times A \rightarrow A$ be a binary operation in A and $x, y \in A$ then $f(x, y)$ is composite of x and y under the composition f .

Laws of binary operations with examples:

(i) closure law: - let a binary operation \circ defined on the non-empty set A then A is closed if $a \circ b \in A$ for all $a, b \in A$.

e.g. The set of natural number (\mathbb{N}) is closed under the binary operation '+'.
ie. Set of integers including zero is associative under ordinary operation addition '+'.
(ii) Associative law: - let a binary operation \circ defined on the non-empty set A . Then A is associative if $a \circ (b \circ c) = (a \circ b) \circ c$ for all $a, b, c \in A$.

(iii) let a binary operation ' \circ ' defined on the non-empty set A . Then A is commutative if $a \circ b = b \circ a$ for all $a, b \in A$.

ie. set of positive integers is commutative with respect to addition '+'.
(iv) Existence of identity element: - let \circ be a binary operation defined on 'a'. Then for a particular element $e \in A$ such that $a \circ e = e \circ a = a$ for all $a \in A$. We say that the identity element exists in A for the binary operation ' \circ '.

ie. let \mathbb{R} be the set of real number and \times is the binary operation called multiplication. 1 is the identity element which exists such that $a \times 1 = 1 \times a = a$ for all $a \in \mathbb{R}$.

ii) Existence of inverse element: - Let \circ be a binary operation defined on 'A'. There exists a corresponding element $a \in A$ such that $a \circ a^{-1} = a^{-1} \circ a = e$. Where e is the identity element of A under the same operation ' \circ '. Then a^{-1} is called inverse of a or a is the inverse of a^{-1} .
 i.e. Let Z be the set of integers and '+' is binary operation called addition then to each $a \in Z$, we have an integer $-a \in Z$ such that $a + (-a) = (-a) + a = 0$.

ii) Distributive law: - Let \circ and $*$ are two binary operations defined over a set A such that

$$a \circ (b * c) = (a \circ b) * (a \circ c) \text{ for all } a, b, c \in A.$$

then the binary operation $*$ is distributive over the operation \circ .

i.e. Let Z be the set of positive integers defined on Z . Then

$$a \times (b + c) = \cancel{a \times b} + a \times c$$

Where $a, b, c \in Z$.